

# An Automated Solver to Determine ML Estimates for the AMSAA Discrete Reliability Growth Model

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## SUMMARY AND CONCLUSIONS

In the tutorial *Reliability Growth Management, Models, and Standards* presented at the 1994 AR&MS, Dr. Larry H. Crow discussed the AMSAA discrete reliability growth model and presented an example problem to illustrate data analysis procedures. Based on reliability testing data, the model requires the user to solve for the maximum likelihood estimates of two parameters,  $\lambda$  and  $\beta$ , needed to determine the reliability growth of the system. Solving for the estimates of these parameters is not a trivial exercise. This paper describes the use of an optimization technique known as evolutionary programming (EP) to determine the estimates of the parameters. The EP optimization algorithm was embedded into an automated spreadsheet solver and used to solve the example problem presented by Crow.

The example problem produced a nonlinear response surface that is difficult to solve for hill-climbing optimizers such as the Excel Solver. The automated EP solver developed by the authors presents a practical and effective solution tool to determine the parameters  $\lambda$  and  $\beta$  in the AMSAA discrete reliability growth model. The EP solver produced estimates of  $\lambda$  and  $\beta$  that were very similar to those provided by Crow.

## 1. INTRODUCTION

### 1.1 AMSAA Discrete Reliability Growth Model

The AMSAA discrete reliability growth model (Refs. 1 and 2) was developed in 1983 by Dr. Larry H. Crow. This model is applicable to single-shot systems and estimates reliability growth on a configuration-by-configuration basis. Configurations are indexed by  $i = 1, 2, \dots, k$ . Using notation as presented in (Ref. 1), let

$T_i$  = cumulative number of trials to the end of configuration  $i$

$N_i$  = number of trials for the  $i$ -th configuration

$$T_i = N_i, T_2 = N_1 + N_2, T_i = \Sigma N_i.$$

$M_i$  = number of failures observed for the  $i$ -th configuration

$f_i$  = failure probability for the  $i$ -th configuration

$R_i$  = reliability for the  $i$ -th configuration,  $R_i = 1 - f_i$ .

The failure probability of the  $i$ -th configuration is given as

$$f_i = \frac{\lambda T_i^\beta - \lambda T_{i-1}^\beta}{N_i} \quad (1)$$

where the parameters  $\lambda > 0$  and  $\beta > 0$  and  $T_0 = 0$ . Based on reliability testing data, the model requires the user to solve for the maximum likelihood estimates of the two parameters  $\lambda$  and  $\beta$  needed to determine the reliability growth of the system.

The maximum likelihood estimates  $\hat{\lambda}$  and  $\hat{\beta}$ , if they exist, are values that satisfy both

$$\sum_{i=1}^k H_i \times S_i = 0 \quad (2)$$

and

$$\sum_{i=1}^k U_i \times S_i = 0 \quad (3)$$

where

$$H_i = [T_i^{\hat{\beta}} \log T_i - T_{i-1}^{\hat{\beta}} \log T_{i-1}] \quad (4)$$

$$U_i = T_i^{\hat{\beta}} - T_{i-1}^{\hat{\beta}} \quad (5)$$

$$S_i = \left[ \frac{M_i}{\hat{\lambda} T_i^{\hat{\beta}} - \hat{\lambda} T_{i-1}^{\hat{\beta}}} - \frac{N_i - M_i}{N_i - \hat{\lambda} T_i^{\hat{\beta}} + \hat{\lambda} T_{i-1}^{\hat{\beta}}} \right]. \quad (6)$$

As presented in (Ref. 1), the reader is left to solve for the maximum likelihood estimates  $\hat{\lambda}$  and  $\hat{\beta}$  needed to determine the reliability  $R_i$  of each configuration of the system.

### 1.2 A Technique to Determine $\lambda$ and $\beta$

William P. Clay (Ref. 2) presents a detailed description of a two dimensional binary search technique to determine the maximum likelihood estimates  $\hat{\lambda}$  and  $\hat{\beta}$ . Using the discrete reliability model, he constructs a likelihood equation for the probability of failure for the  $i$ -th configuration. Values for the maximum likelihood estimates are found that maximizes the



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